The Expansion of Krugman-Flood-Garber Model

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Abstract: The Krugman-Flood-Garber model was based on the interest rate parity theory. The interest rate parity theory is based on the marketization of interest rate and full convertibility of currency, so the interest rate parity was not applicable to those countries that impose capital control. In this paper, considering variable system friction factors, we establish a new currency crisis model based on the Krugman-Flood-Garber model. The new model is more general. And then, we obtain the calculation equation of the collapse of fixed exchange rate time.

1. Introduction

Krugman (1979) established the speculative shock model at a fixed exchange rate based on the model of the speculative impact on gold prices established by Federal Reserve Bank economist Steven Salant & Dale Henderson (1978), which was also the earliest theoretical model of the currency crisis. Since Krugman's (1979) crisis theory model was a non-linear model and it had no explicit solution, Flood and Garber (1984a) simplified Krugman's (1979) model with a linear model and obtained an explicit solution to the speculative attack time. The academic community regarded Krugman-Flood-Garber as the first generation standardized model of the currency crisis theory.

Subsequently, many scholars did some deeper study based on the KFG model. Flood and Garber (1984) studied the assumption of uncertainty about the growth of domestic credit in the basic crisis model. In their model, they think the growth rate of domestic credit was random, so the time of a fixed exchange rate crash was also random. After Flood and Garber established this stochastic method, many scholars used this model to conduct empirical research. Blanco and Garber (1986), Connolly & Fernandéz (1987) and Goldberg (1990) studied the Mexican crisis. Cumby & Wijbergen (1987) studied the Argentine currency crisis and Grilli (1989) studied the situation of the United States from 1894 to 1896.

In the Krugman-Flood-Garber model, assuming that the fixed exchange rate collapses, the exchange rate regime becomed completely floating, but in fact, there were many options available. Studies by Blanco and Garbe (1986), Grilli (1986) and Wyplosz (1986) argued that currency devaluations could be made. Dornbush (1987) argued that a two-track system could be implemented .He also argued that it was not necessary that the crawling peg system would be implemented after the collapse of the fixed exchange rate or the exhaustion of foreign exchange reserves

Guo Qingma (2009), a Chinese scholar, extended the Krugman-Flood-Garber modell. The creative thinking in his research was the introduction of the "system friction". The model was revised so as to make the model more in line with the reality of financially immature countries, not fully marketized interest rates and the existence of capitalist countries such as China.

Based on the research of Krugman-Flood-Garber's standardization model and the Guo's model, this paper introduces "time-varying friction coefficient and further expand the model .The model established in this paper has more general significance.

2. KFG standardized model and extended model

2.1 KFG model

The KFG model introduced five basic equations as frameworks, which respectively represented the equilibrium of money market, the money supply mechanism, the fixed growth of domestic

credit, the purchasing power parity and the interest rate parity, as follows:

$$\frac{M(t)}{P(t)} = a_0 - a_1 r(t), \, a_0, a_1 > 0 \tag{1}$$

$$M(t) = R(t) + D(t) \tag{2}$$

$$\hat{D}(t) = \mu \tag{3}$$

$$P(t) = P^*(t)S(t) \tag{4}$$

$$r(t) = r^*(t) + \hat{S}(t)/S(t) \tag{5}$$

Equation (1) is the equilibrium of the domestic money market, the left side of the equation is the money supply, and the right side of the equation is the demand of money $M(t)$, $P(t)$ versus $r(t)$ respectively t Time base domestic currency, domestic price level and interest rate level. Equation (2) for the money supply mechanism equation, among them $M(t)$, $R(t)$ with $D(t)$ Respectively t Time high-energy money supply, domestic credit and international reserves. In the equation (3), $\hat{D}(t)$ was for the domestic credit growth rate, μ was ac constant. Equation (4) was the purchasing power parity expression, among them, $P(t)$ the domestic price level, $P^*(t)$ the level of foreign prices, $S(t)$ the spot exchange rate. Equation (5) was for the parity of interest rates, $r(t)$ the domestic and international interest rates, $r^*(t)$ respectively, said the level of foreign interest rates. According to the model, we could calculate the time when the fixed exchange rate collapsed after the crisis, the domestic credit when the fixed exchange rate collapsed and the foreign exchange reserve level when the fixed exchange rate collapsed.

Fixed exchange rate crash time T:

$$T = R(t_0)/\mu - \frac{\theta}{\lambda} \tag{6}$$

Domestic credit prior to the collapse of the fixed exchange rate could be expressed as follows:

$$D(t) = D(t_0) + \mu t \tag{7}$$

Domestic credit at a fixed exchange rate crashed:

$$D(T) = D(t_0) + R(t_0) - \frac{\theta\mu}{\lambda} \tag{8}$$

when the fixed exchange rate crashed, the money supply was:

$$M(T) = D(t_0) + R(t_0) \tag{9}$$

The foreign exchange reserves at the time of a fixed exchange rate collapse was:

$$R(T) = \frac{\theta\mu}{\lambda}$$

2.2 Model expansion considering fixed institutional friction factors

The interest rate parity condition were a basic premise of Krugman-Flood-Garber Model. The basic premise of interest rate parity was the marketability of interest rates and the complete convertibility of currencies. However, in many countries with underdeveloped capital controls on some financial markets, theirs interests rate were not basically determined by the market and capital flows were limited; the exchange rate was inelastic. China is a typical case. Based on this consideration, Guo (2009), a Chinese scholar, introduced the coefficient of institutional friction and

established a deformed interest rate parity model, so as to correct the standardized crisis model. According to this model, the point when a fixed exchange rate system collapsed in a crisis economy was calculated. The results showed that the friction coefficient delayed the collapse of the fixed exchange rate system.The expansion model was as follows:

$$r = r^* + \frac{(\tilde{S} - S)}{S} \pm (C + K) \tag{10}$$

Among them, r and r^* respectively said the domestic interest rates and foreign interest rates. "+" said that the flow of arbitrage capital to the country. "-" said the flow of arbitrage capital abroad. S was the spot exchange rate. $\tilde{S}$ That the expected exchange rate (because there was no perfect forward foreign exchange market in China, all with the expected exchange rate instead of the forward exchange rate); C was for transaction costs and K for the system of friction coefficient. Following was the framework of the standardization model of the first generation of currency crisis theory:

$$\frac{M(t)}{P(t)} = a_0 - a_1 r(t), \quad a_0, a_1 > 0 \tag{11}$$

$$M(t) = R(t) + D(t) \tag{12}$$

$$\dot{D}(t) = \mu, \quad \mu > 0 \tag{13}$$

$$P(t) = P^*(t)S(t) \tag{14}$$

$$r = r^* + \frac{\tilde{S}(t) - S(t)}{S(t)} - (C + K) \tag{15}$$

The economic variables in the above models (11) to (15) had the same meaning as the economic variables in (1) to (5). The fixed exchange rate crash time based on the modified model could be calculated:

$$T_1 = \frac{R(t_0)}{\mu} - \frac{\theta_1}{\lambda_1} \tag{16}$$

Among them $R(t_0)$ said the initial moment of foreign exchange reserves, and $\lambda_1 = a_0 P^* - a_1 P^*[r^* - (C + K)]$, $\theta_1 = a_1 P^*$. Comparison (15) and (16), easy to know $\theta_1 = \theta$, $\lambda_1 > \lambda$, so $T_1 > T$. This showed that the currency exchange rate to collapse would be delayed because of the speculative attacks due to transaction costs and institutional friction. The increase of transaction costs C and the system of friction coefficient K would lead to a corresponding extension of the fixed exchange rate maintenance time.

3. The expansion of the model

The above model assumed that the coefficient of institutional friction was a constant, that was to say it assumed that the monetary authorities had not changed their capital controls and efforts in response to the crisis. I fact, we would not doubt that the monetary authorities would choose the means and intensity of capital controls based on their own foreign exchange reserves and the status quo of the international capital flows in response to the currency crisis . They would always try its best to stop the deterioration of the crisis while trying not to undermine the country's financial order. For example, at the time the authorities judged the crisis was within its control, the efforts to control capital flows may be small because they do not affect the normal flow of capital between their home countries.   In this paper, we would establish a model based on the above assumption.

We evolved the interest parity condition from (15) to

$$r = r^* + \frac{\tilde{S}(t) - S(t)}{S(t)} - [C + K(t)] \qquad (17)$$

In the equation (17), the meanings of other variables were the same as those of the variables in (15) except $K(t)$. $K(t)$ was defined as a time-dependent variable rather than a constant, which was based on the strength of the monetary authorities in implementing measures and instruments such as interest rate controls and capital flow controls in response to the currency crisis.

Based on this interest rate parity, we established a currency crisis model:

$$\frac{M(t)}{P(t)} = a_0 - a_1 r(t), \quad a_0, a_1 > 0 \qquad (18)$$

$$M(t) = R(t) + D(t) \qquad (19)$$

$$\dot{D}(t) = \mu, \quad \mu > 0 \qquad (20)$$

$$P(t) = P^*(t) S(t) \qquad (21)$$

$$r = r^* + \frac{\tilde{S}(t) - S(t)}{S(t)} - [C + K(t)] \qquad (22)$$

We could know:

$$r(t) - r^*(t) = (\tilde{S}(t) - S(t)) / S(t) - (C + K(t)) \qquad (23)$$

If $X(t) = r(t) - r^*(t)$, we could know:

$$\frac{dX(t)}{dt} = \frac{d}{dt}\left(\frac{\tilde{S} - S}{S}\right) - \frac{dK(t)}{dt} \qquad (24)$$

In (24), t is for time. We suppose the exchange rate convexity was constant, that was to say we assumed $\frac{d}{dt}\left(\frac{\tilde{S} - S}{S}\right)$ was a constant:

$$\frac{d}{dt}\left(\frac{\tilde{S} - S}{S}\right) = E \qquad (25)$$

In equation (25), E was a constant. We defined β as damping coefficient:

$$\frac{dK(t)}{dt} = \beta X(t) \qquad (26)$$

From (24), we could get:

$$X(t) + K(t) = Et + C^* \qquad (27)$$

C^* was a constant.
Based on (24) and (26), we got:

$$\frac{dX(t)}{dt} + \beta X(t) = E \qquad (28)$$

Given the initial conditions: $X\big|_{t=t_0} = X_0$, the equation (28) could be solved:

$$X(t) = X_0 e^{-\beta(t-t_0)} + \frac{E}{\beta}(1 - e^{-\beta(t-t_0)}) \tag{29}$$

Based on equation (27) and (28), we would get $K(t)$:

$$K(t) = Et + (\frac{E}{\beta} - X_0)e^{-\beta(t-t_0)} - \frac{E}{\beta} + C^* \tag{30}$$

According to the same method mentioned above, we could get the fixed exchange rate crash time:

$$T_2 = \frac{R(t_0)}{\mu} - \frac{\theta_2}{\lambda_2} \tag{31}$$

In the equation, $\lambda_2 = a_0 p^* - a_1 p^*[r^* - (C + K(t))]$. If $T_2 = t$, we could get:

$$\vartheta t^2 + \xi t + \eta e^{-\beta(t-t_0)}t + \varepsilon e^{-\beta(t-t_0)} + \delta = 0 \tag{32}$$

Among them,

$$\vartheta = a_1 p^* E \tag{33}$$

$$\xi = a_0 p^* - a_1 p^* r^* + a_1 p^* c - a_1 p^* \frac{E}{\beta} + a_1 p^* c^* - \frac{E}{\mu} a_1 p^* R(t_0) \tag{34}$$

$$\eta = a_1 p^* (\frac{E}{\beta} - X_0) \tag{35}$$

$$\varepsilon = -\frac{1}{\mu} a_1 p^* R(t_0)(\frac{E}{\beta} - X_0) \tag{36}$$

$$\delta = a_1 p^* - \frac{R(t_0)(a_0 p^* - a_1 p^* r^* + a_1 p^* c + a_1 p^* c^* - a_1 p^* \frac{E}{\beta})}{\mu} \tag{37}$$

Based on mathematical knowledge, it was easy to know t will change in the light of changes in the authorities' adjustment of the timing of monetary market interventions. It was conceivable that if the authorities change the means of intervention in respond to the situation of the currency crisis, the collapse of the fixed exchange rate system may be considerably delayed. On the contrary, if the authorities did not correctly judge the stage of development of the crisis at some point, and they changed the control strategy at the wrong point, the authorities would not only faile to intervene the market to delay the collapse of the exchange rate system, but also it may even speed up the crisis.

4. Conclusion

The Krugman-Flood-Garber standardization model was based on the basic premise of the establishment of interest rate parity, and the precondition for the establishment of interest rate parity was the marketization of interest rates and the complete convertibility of currency. However, in many countries with underdeveloped capital controls on some financial markets, interest rate was not marketized and money was not exchanged freely. We believe that the existence of transaction costs and institutional frictions delay the collapse of the currency exchange rate under the speculative attacks, and the increase of transaction costs and system friction will lead to a corresponding prolongation of the fixed exchange rate maintenance. This paper argues that the coefficient of institutional friction is not a constant. In fact, in response to the currency crisis, monetary authorities will choose the means and intensity of capital controls based on their own foreign exchange reserves and the status quo of domestic and international capital flows. Based on

this, we establishes an extended model of Krugman-Flood-Garber in this paper, and the solution of fixed exchange rate collapse time is given. Unfortunately, this equation is extremely complicated. Therefore, no calculation is made about the fixed exchange rate crash time .This will be a focus of our ongoing research.

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